Nonlinear phenomena of generation of longitudinal electric current by transversal electromagnetic field in plasmas

A. V. Latyshev[1] and A. A. Yushkanov[2]

Faculty of Physics and Mathematics,
Moscow State Regional University, 105005,
Moscow, Radio str., 10A

Abstract

The analysis of nonlinear interaction of transversal electromagnetic field with collisionless plasma is carried out. Formulas for calculation electric current in collisionless plasma with arbitrary degree of degeneration of electronic gas are deduced. It has appeared, that the nonlinearity account leads to occurrence of the longitudinal electric current directed along a wave vector. This second current is orthogonal to the known transversal current, received at the classical linear analysis.

Key words: collisionless plasmas, Vlasov equation, Dirac, Fermi, plasma with arbitrary degree of degeneration of electronic gas, electrical current.

PACS numbers: 52.25.Dg Plasma kinetic equations, 52.25.-b Plasma properties, 05.30 Fk Fermion systems and electron gas

Introduction

Dielectric permeability in quantum plasma was studied by many authors [1] – [11]. Dielectric permeability is one of the major plasma characteristics.

This quantity is necessary for the description of skin-effect [12], for the analysis surface plasmons [13], for descriptions of process of propagation and attenuation of the transversal plasma oscillations [8], for studying of the mechanism of penetration electromagnetic waves in plasma [7], and for the analysis of other problems in the plasma physics [14] – [19].

[1] *avlatyshev@mail.ru*
[2] *yushkanov@inbox.ru*

Let us notice, that for the first time in work [1] the formula for calculation of longitudinal dielectric permeability into quantum plasma has been deduced. Then the same formula has been deduced and in work [2].

In the present work formulas for calculation electric current into collisionless plasma at any temperature (at any degrees of degeneration of the electronic gas) are deduced.

It has appeared, that electric current expression consists of two summands. The first summand, linear on vector potential, is known classical expression of electric current. This electric current is directed along vector potential of electromagnetic field. The second summand represents itself electric current, which is proportional to the square vector potential of electromagnetic field. The second current is perpendicular to the first and it is directed along the wave vector. Occurrence of the second current comes to light the spent account nonlinear character interactions of electromagnetic field with plasma.

Let us underline, that the nonlinear phenomena in plasma are studied more half century (see, for example, [20] - [22]). However, the electric current directed along a wave vector, was not is revealed earlier.

1. Vlasov kinetic equation and its solution

Let us consider Vlasov equation describing behaviour of collisionless plasmas

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}+e\Big(\mathbf{E}+\frac{1}{c}[\mathbf{v},\mathbf{H}]\Big)\frac{\partial f}{\partial \mathbf{p}}=0. \tag{1.1}$$

Vector potential we take as orthogonal to direction of a wave vector $\mathbf{k}$:

$$\mathbf{k}\mathbf{A}(\mathbf{r},t)=0. \tag{1.2}$$

in the form of the running harmonious wave

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}_0e^{i(\mathbf{k}\mathbf{r}-\omega t)}.$$

Scalar potential we will consider equal to zero. Electric and magnetic fields are connected with vector potential by equalities

$$\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},\qquad \mathbf{H}=\operatorname{rot}\mathbf{A}. \tag{1.3}$$

The wave vector we direct along axis x: $\mathbf{k}=k(1,0,0)$, and vector potential of elec tromagnetic field we direct along axis y:

$$\mathbf{A}=A_y(x,t)(0,1,0),\qquad A_y(x,t)\sim e^{i(kx-\omega t)}.$$

Then

$$A_y=-\frac{ic}{\omega}E_y,\qquad \mathbf{H}=\frac{ck}{\omega}E_y(0,0,1),$$
$$[\mathbf{v},\mathbf{H}]=\frac{ck}{\omega}E_y(v_y,-v_x,0).$$

Let us operate with method of consecutive approximations. Considering, that the member with an electromagnetic field has an order, on unit smaller other members, let us rewrite the equation (1.1) in the form

$$\frac{\partial f^{(k)}}{\partial t}+v_x\frac{\partial f^{(k)}}{\partial x}+$$
$$+eE_y\left(\frac{\partial f^{(k-1)}}{\partial p_y}\Big(1-\frac{kv_x}{\omega}\Big)+\frac{kv_y}{\omega}\frac{\partial f^{(k-1)}}{\partial p_x}\right)=0,\quad k=1,2. \tag{1.4}$$

Here in zero approximation $f^{(0)}$ is the absolute Fermi—Dirac distribution,

$$f^{(0)}=f_0=\Big[1+\exp\Big(\frac{mv^2}{2k_BT}-\frac{\mu}{k_BT}\Big)\Big]^{-1},$$

k_B is the Boltzmann constant, T is the plasmas temperature, μ is the chemical potential of plasmas.

It is easy to see, that

$$\frac{mv^2}{2k_BT}-\frac{\mu}{k_BT}=\frac{\mathcal{E}}{\mathcal{E}_T}-\alpha=P^2-\alpha,$$

where α is the dimensionless (normalized) chemical potential, $P = v/v_T = p/p_T$ is the dimensionless electron velosity (or momentum),

$$\mathcal{E} = \frac{mv^2}{2}, \quad \mathcal{E}_T = \frac{mv_T^2}{2}, \quad v_T = \sqrt{\frac{2k_BT}{m}}, \quad \alpha = \frac{\mu}{k_BT}.$$

Therefore in zero approximation

$$f_0(P) = \frac{1}{1 + e^{P^2 - \alpha}}.$$

We notice that

$$[\mathbf{v}, \mathbf{H}]\frac{\partial f^{(0)}}{\partial \mathbf{p}} = 0,$$

because

$$\frac{\partial f^{(0)}}{\partial \mathbf{p}} \sim \mathbf{v}.$$

We search solution in first approximation in the form

$$f^{(1)} = f^{(0)} + f_1(x, t, P_x).$$

In this approximation the equation (1.4) becomes simpler

$$\frac{\partial f_1}{\partial t} + v_x\frac{\partial f_1}{\partial x} = -eE_y\frac{\partial f^{(0)}}{\partial p_y}. \tag{1.5}$$

From the equation (1.5) it is found

$$f_1 = \frac{2ieE_y}{p_T}\frac{P_y g(P)}{\omega - kv_T P_x}, \tag{1.6}$$

where

$$g(P) = \frac{e^{P^2-\alpha}}{(1 + e^{P^2-\alpha})^2}.$$

In the second approximation for function $f^{(2)}$ we search in the form

$$f^{(2)} = f^{(1)} + f_2(x, t, v_x), \quad \text{where} \quad f_2 \sim E_y^2(x, t).$$

Let us substitute $f^{(2)}$ in (1.4). We receive the equation

$$\frac{\partial f_2}{\partial t} + v_x\frac{\partial f_2}{\partial x} + eE_y\frac{\partial f_1}{\partial p_y} + \frac{e}{c}[\mathbf{v}, \mathbf{H}]\frac{\partial f_1}{\partial \mathbf{p}} = 0.$$

From this equation it is found

$$f_2=\frac{e^2E_y^2}{p_T^2(\omega-kv_TP_x)}\Bigg[\frac{\dfrac{\partial}{\partial P_y}\Big(P_yg(P)\Big)}{\omega-kv_TP_x}+\frac{kv_T}{\omega}P_y^2\frac{\partial}{\partial P_x}\Big(\frac{g(P)}{\omega-kv_TP_x}\Big)-$$

$$-\frac{kv_T}{\omega}P_x\frac{\dfrac{\partial}{\partial P_y}\Big(P_yg(P)\Big)}{\omega-kv_TP_x}\Bigg]. \qquad (1.7)$$

Distribution function in square-law approximation on the field it is constructed

$$f=f^{(2)}=f^{(0)}+f_1+f_2, \qquad (1.8)$$

where f_1, f_2 are given accordingly by formulas (1.6) and (1.7).

2. Density of electric current

Let us calculate current density

$$\mathbf{j}=e\int \mathbf{v}f\frac{2d^3p}{(2\pi\hbar)^3}. \qquad (2.1)$$

By means of (1.8) it is visible, that the vector of current density has two nonzero components

$$\mathbf{j}=(j_x,j_y,0).$$

Here j_y is the density of known transversal current, calculated as

$$j_y=e\int v_yf\frac{2d^3p}{(2\pi\hbar)^3}=e\int v_yf_1\frac{2d^3p}{(2\pi\hbar)^3}. \qquad (2.2)$$

This current is directed along electric field, its density is deduced by means of linear approximation of distribution function.

Square-law on quantity of an electromagnetic field composed f_2 the contribution to density of a current does not bring. Density of transversal current it is calculated under the formula

$$j_y=\frac{ie^2k_T^3}{2\pi^3m}E_y(x,t)\int\frac{P_y^2g(P)d^3P}{\omega-kv_TP_x}. \qquad (2.3)$$

Here k_T is the thermal wave number,

$$k_T=\frac{p_T}{\hbar}=\frac{mv_T}{\hbar}.$$

Let us calculate the longitudinal current. For density of longitudinal current according to definition it is had

$$j_x=e\int v_xf\frac{2d^3p}{(2\pi\hbar)^3}=e\int v_xf_2\frac{2d^3p}{(2\pi\hbar)^3}=$$
$$=\frac{2ev_Tp_T^3}{(2\pi\hbar)^3}\int P_xf_2d^3P.$$

Having taken advantage (1.7), from here we receive, that

$$j_x=e^3E_y^2\frac{2mv_T^2}{(2\pi\hbar)^3}\int\Bigg[\frac{\dfrac{\partial}{\partial P_y}\Big(P_yg(P)\Big)}{\omega-kv_TP_x}+\frac{kv_T}{\omega}P_y^2\frac{\partial}{\partial P_x}\Big(\frac{g(P)}{\omega-kv_TP_x}\Big)-$$
$$-\frac{kv_T}{\omega}P_x\frac{\dfrac{\partial}{\partial P_y}\Big(P_yg(P)\Big)}{\omega-kv_TP_x}\Bigg]\frac{P_xd^3P}{\omega-kv_TP_x}. \tag{2.4}$$

Equality (2.4) can be simplified

$$j_x=e^3E_y^2\frac{2mv_T^2}{(2\pi\hbar)^3}\int\Bigg[\frac{1}{\omega}\frac{\partial}{\partial P_y}(P_yg(P))+$$
$$+\frac{kv_T}{\omega}P_y^2\frac{\partial}{\partial P_x}\Big(\frac{g(P)}{\omega-kv_TP_x}\Big)\Bigg]\frac{P_xd^3P}{\omega-kv_TP_x}. \tag{2.5}$$

The first integral from (2.5) is equal to zero. Really, we will consider internal integral on P_y

$$\int\limits_{-\infty}^{\infty}\frac{\partial}{\partial P_y}(P_yg(P))dP_y=P_yg(P)\Bigg|_{P_y=-\infty}^{P_y=+\infty}=0.$$

Now in the second integral from (2.5) we will calculate internal integral on P_x

$$\int\limits_{-\infty}^{\infty}\frac{\partial}{\partial P_x}\Big(\frac{g(P)}{\omega-kv_TP_x}\Big)\frac{P_xdP_x}{\omega-kv_TP_x}=$$

$$= \frac{g(P)P_x}{(\omega - kv_TP_x))^2}\Bigg|_{P_x=-\infty}^{P_x=+\infty} - \int\limits_{-\infty}^{\infty}\frac{g(P)}{\omega - kv_TP_x}d\Big(\frac{P_x}{\omega - kv_TP_x}\Big) =$$

$$= -\omega\int\limits_{-\infty}^{\infty}\frac{g(P)dP_x}{(\omega - kv_TP_x)^3}.$$

Thus, equality (2.5) becomes simpler

$$j_x = -e^3E_y^2\frac{2mv_T^3k}{(2\pi\hbar)^3}\int\frac{g(P)P_y^2d3P}{(\omega - kv_TP_x)^3} =$$

$$= \frac{e^3E_y^2}{4\pi^3\hbar mv_T^2q^2}\int\frac{g(P)P_y^2d^3P}{(P_x - \Omega/q)^3} = \frac{e^3E_y^2q}{4\pi^3\hbar mv_T^2}\int\frac{g(P)P_y^2d^3P}{(qP_x - \Omega)^3}. \qquad (2.6)$$

Here

$$\Omega = \frac{\omega}{k_Tv_T}, \qquad q = \frac{k}{k_T}.$$

Double internal integral from (2.6) in plane (P_y, P_z) it is calculated in polar coordinates ($P_y = \rho\cos\varphi, P_z = \rho\sin\varphi$):

$$\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}g(P)P_y^2dP_ydP_z = \int\limits_{0}^{2\pi}\int\limits_{0}^{\infty}\frac{\cos^2\varphi e^{P^2-\alpha}\rho^3}{(1+e^{P^2-\alpha})^2}d\varphi d\rho =$$

$$= \frac{\pi}{2}\ln(1+e^{\alpha-P_x^2}), \quad P^2 = \rho^2 + P_x^2.$$

Hence, the density of longitudinal current is equal to

$$j_x = \frac{e^3E_y^2q}{4\pi^2\hbar mv_T^2q^2}\int\limits_{-\infty}^{\infty}\frac{\ln(1+e^{\alpha-P_x^2})dP_x}{(P_x - \Omega/q)^3} = \qquad (2.7)$$

$$= \frac{e^3E_y^2q}{4\pi^2\hbar mv_T^2}\int\limits_{-\infty}^{\infty}\frac{\ln(1+e^{\alpha-P_x^2})dP_x}{(qP_x - \Omega)^3}, \qquad (2.8)$$

or

$$j_x = \frac{e^3E_y^2}{4\pi^2\hbar mv_T^2q^2}\int\limits_{-\infty}^{\infty}\ln(1+e^{\alpha-(P_x+\Omega/q)^2})\frac{dP_x}{P_x^3}.$$

Let us present the formula (2.8) in an invariant form. For this purpose we will introduce transversal electric field

$$\mathbf{E}_{\text{tr}}=\mathbf{E}-\frac{\mathbf{k}(\mathbf{E}\mathbf{k})}{k^2}=\mathbf{E}-\frac{\mathbf{q}(\mathbf{E}\mathbf{q})}{q^2}.$$

Now equality (2.8) we will present in coordinate-free form

$$\mathbf{j}^{\text{long}}=\frac{e^3\mathbf{E}_{\text{tr}}^2\mathbf{q}}{4\pi^2\hbar m v_T^2}\int\limits_{-\infty}^{\infty}\frac{\ln(1+e^{\alpha-\tau^2})d\tau}{(q\tau-\Omega)^3}.$$

The integral (2.7) is calculated with use of known rule of Landau as Cauchy type integral

$$j_x=\frac{e^3E_y^2}{4\pi^2\hbar m v_T^2}q\Bigg[-i\pi\frac{1}{2q^3}\Big(\ln(1+e^{\alpha-\tau^2})\Big)''\Bigg|_{\tau=\Omega/q}+$$

$$+\text{V.p.}\int\limits_{-\infty}^{\infty}\frac{\ln(1+e^{\alpha-\tau^2})d\tau}{(q\tau-\Omega)^3}\Bigg].$$

Symbol V.p. means principal value of integral. The sign choice $x-i\varepsilon$ in the previous formula means attenuation in time of potential of an electromagnetic field

$$\mathbf{A}_0e^{i(\mathbf{kr}-(\omega-i\varepsilon)t)}=\mathbf{A}_0e^{-\varepsilon t}e^{i(\mathbf{kr}-\omega t)}\to 0,\qquad \forall\varepsilon>0.$$

We will present this formula in the following form

$$\mathbf{j}^{\text{long}}=\sigma_{\text{l,tr}}\mathbf{E}_{\text{tr}}^2\mathbf{q}J(\Omega,q),$$

where $J(\Omega,q)$ is the dimensionless part of electric current density,

$$J(\Omega,q)=-i\frac{\pi}{2q^3}\Big[\ln(1+e^{\alpha-\tau^2})\Big]''\Bigg|_{\tau=\Omega/q}+\text{V.p.}\int\limits_{-\infty}^{\infty}\frac{\ln(1+e^{\alpha-\tau^2})d\tau}{(q\tau-\Omega)^3},$$

$$\sigma_{\rm l,tr}=\frac{e^3}{4\pi^2\hbar m v_T^2}=\frac{e^3k_BT}{4\pi^2\hbar m^2}.$$

On fig. 1-3 we will present graphics of behaviour of the real part of dimensionless quantity of density of electric current

$$\mathrm{Re}(J(\Omega,q))=\mathrm{V.p.}\int\limits_{-\infty}^{\infty}\frac{\ln(1+e^{\alpha-\tau^2})d\tau}{(q\tau-\Omega)^3}.$$

On fig. 4,5 we will present graphics of behaviour of the imaginary part of dimensionless quantity of density of electric current,

$$\mathrm{Im}\,(J(\Omega,q))=-\frac{\pi}{2q^3}\Big[\ln(1+e^{\alpha-\tau^2})\Big]''\Bigg|_{\tau=\Omega/q}=$$

$$=\pi\frac{(1-2\tau^2)e^{\alpha-\tau^2}+e^{2(\alpha-\tau^2)}}{q^3(1+e^{\alpha-\tau^2})^2}\Bigg|_{\tau=\Omega/q}=$$

$$=\pi\frac{1+(1-2\tau^2)e^{\tau^2-\alpha}}{q^3(1+e^{\tau^2-\alpha})^2}\Bigg|_{\tau=\Omega/q}.$$

In case of small values of wave number from (2.6) it is received

$$j_x=-\frac{e^3E_y^2(x,t)}{2\pi^2\hbar m v_T^2\Omega^3}\cdot q\cdot\int\limits_0^{\infty}\ln(1+e^{\alpha-\tau^2})d\tau.$$

It is possible to present this equality in the form

$$j_x=-\frac{2e^3E_y^2(x,t)\mathcal{E}_T}{\pi^2\hbar(\hbar\omega)^3}\cdot q\cdot\int\limits_0^{\infty}\ln(1+e^{\alpha-\tau^2})d\tau.$$

Let us find numerical density (concentration) of plasma in the equilibrium condition

$$N=\int f_0(P)\frac{2d^3p}{(2\pi\hbar)^3}=\frac{8\pi p_T^3}{(2\pi\hbar)^3}\int\limits_0^{\infty}\frac{e^{\alpha-P^2}P^2dP}{1+e^{\alpha-P^2}}=$$

$$= \frac{k_T^3}{2\pi^2} \int\limits_0^\infty \ln(1 + e^{\alpha - P^2}) dP.$$

Integral from expression for current density we will express through numerical concentration of plasma in equilibrium condition. It is as a result received, that

$$j_x = -\frac{eE_y^2(x,t)}{4\pi(\hbar\omega)} \frac{\omega_p^2}{\omega^2} q = -\frac{eE_y^2(x,t)}{4\pi m v_T \omega} \frac{\omega_p^2}{\omega^2} \cdot k.$$

Here ω_p is the plasma (Langmuir) frequency,

$$\omega_p = \sqrt{\frac{4\pi e^2 N}{m}}.$$

In coordinate-free form last equality rewritten as follows

$$\mathbf{j}^{\text{long}} = -\frac{e\mathbf{E}_{tr}^2}{4\pi(\hbar\omega)} \frac{\omega_p^2}{\omega^2} \mathbf{q}.$$

3. Conclusions

In the present work the solution of Vlasov equation is used for collisionless plasmas. For the solution it is used the method of consecutive approximations.

As small parametre the quantity of the vector potential of electromagnetic field (or to it proportional quantity of intensity of electric field) is considered.

At use of approximation of the second order it appears, that the electromagnetic field generates an electric current directed along the wave vector, and proportional to the size square of electric field.

Thus an electric current directed along electric field, the same, as in the linear analysis.

REFERENCES

[1] *Klimontovich Y. and Silin V.P.* The Spectra of Systems of Interacting Particles // JETF (Journal Experimental Theoreticheskoi Fiziki), **23**, 151 (1952).

[2] *Lindhard J.* On the properties of a gas of charged particles // Kongelige Danske Videnskabernes Selskab, Matematisk–Fysiske Meddelelser. V. 28, №8 (1954), 1–57.

[3] *Kliewer K.L. and Fuchs R.* Lindhard Dielectric Functions with a Finite Electron Lifetime // Phys. Rev. 1969. V. 181. №2. P. 552–558.

[4] *Mermin N. D.* Lindhard Dielectric Functions in the Relaxation–Time Approximation // Phys. Rev. B. 1970. V. 1, №5. P. 2362–2363.

[5] *Anderson D., Hall B., Lisak M., and Marklund M.* Statistical effects in the multistream model for quantum plasmas // Phys. Rev. E **65** (2002), 046417.

[6] *De Andrés P., Monreal R., and Flores F.* Relaxation–time effects in the transverse dielectric function and the electromagnetic properties of metallic surfaces and small particles // Phys. Rev. **B**. 1986. Vol. 34,№10, 7365–7366.

[7] *Shukla P. K. and Eliasson B.* Nonlinear aspects of quantum plasma physics // Uspekhy Fiz. Nauk, **53**(1) 2010; [V. 180. No. 1, 55-82 (2010) (in Russian)].

[8] *Eliasson B. and Shukla P. K.* Dispersion properties of electrostatic oscillations in quantum plasmas // arXiv:0911.4594v1 [physics.plasm-ph] 24 Nov 2009, 9 pp.

[9] *Tatarskii V. I.* The Wigner representation of quantum mechanics / Uspekhy Fiz. Nauk. **26** (1983), 311–327; [Usp. Fis. Nauk. **139** (1983), 587 (in Russian)].

[10] *Hillery M., O'Connell R. F., Scully M. O., and Wigner E. P.* Distribution functions in physics: Fundamentals // Phys. Rev. **106** (1984), 121–167.

[11] *Dressel M. and Grüner G.* Electrodynamics of Solids. Optical Properties of Electrons in Matter // Cambridge. Univ. Press. 2003. 487 p.

[12] *Gelder van, A. P.*Quantum Corrections in the Theory of the Anomalous Skin Effect // Phys. Rev. 1969. Vol. 187. №3. P. 833–842.

[13] *Fuchs R. and Kliewer K. L.* Surface plasmon in a semi–infinite free–electron gas // Phys. Rev. B. 1971. V. 3. №7. P. 2270–2278.

[14] *Brodin G., Marklund M., Manfredi G.* Quantum Plasma Effects in the Classical Regime // Phys. Rev. Letters. **100**, (2008). P. 175001-1 – 175001-4.

[15] *Manfredi G. and Haas F.* Self-consistent fluid model for a quantum electron gas // Phys. Rev. B **64** (2001), 075316.

[16] *Latyshev A. V. and Yushkanov A. A.* Transverse Electric Conductivity in Collisional Quantum Plasma// Plasma Physics Report, 2012, Vol. 38, No. 11, pp. 899–908.

[17] *Latyshev A. V. and Yushkanov A. A.* Transverse electrical conductivity of a quantum collisional plasma in the Mermin approach // Theor. and Math. Phys., **175**(1): 559–569 (2013).

[18] *Latyshev A. V. and Yushkanov A. A.* Longitudinal Dielectric Permeability of a Quntum Degenerate Plasma with a Constant

Collision Frequency// High Temperature, 2014, Vol. 52, №1, pp. 128–128.

[19] *Latyshev A. V. and Yushkanov A. A.* Longitudinal electric conductivity in a quantum plasma with a variable collision frequency in the framework of the Mermin approach// Theor. and Mathem. Physics, **178**(1): 131-142 (2014).

[20] *Ginsburg V.L., Gurevich A.V.* The nonlinear phenomena in the plasma which is in the variable electromagnetic field//Uspekhy Fiz. Nauk, **70**(2) 1960; p. 201-246 (in Russian).

[21] *Zytovich V.N.* Nonlinear effects in plasmas// Uspekhy Fiz. Nauk, **90**(3) 1966; p. 435-489 (in Russian).

[22] *Zytovich V.N.* Nonlinear effects in plasmas. Moscow. Publ. Leland. 2014. 287 p. (in Russian).

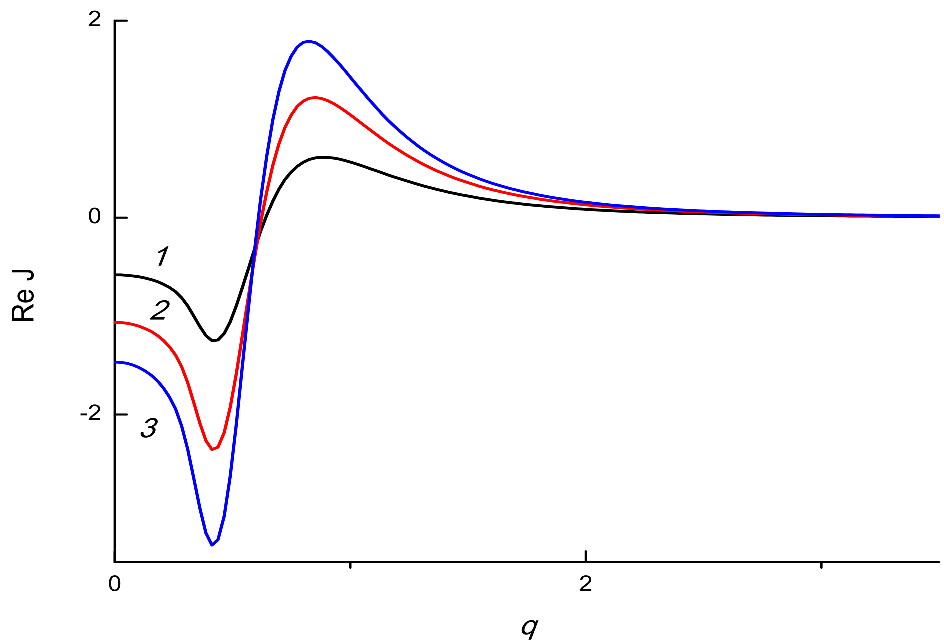

Fig. 1. Real part of longitudinal electric current density, $\Omega = 1$. Curves $1, 2, 3$ correspond to values of dimensionless chemical potential $\alpha = -1, -0.3, +0.1$.

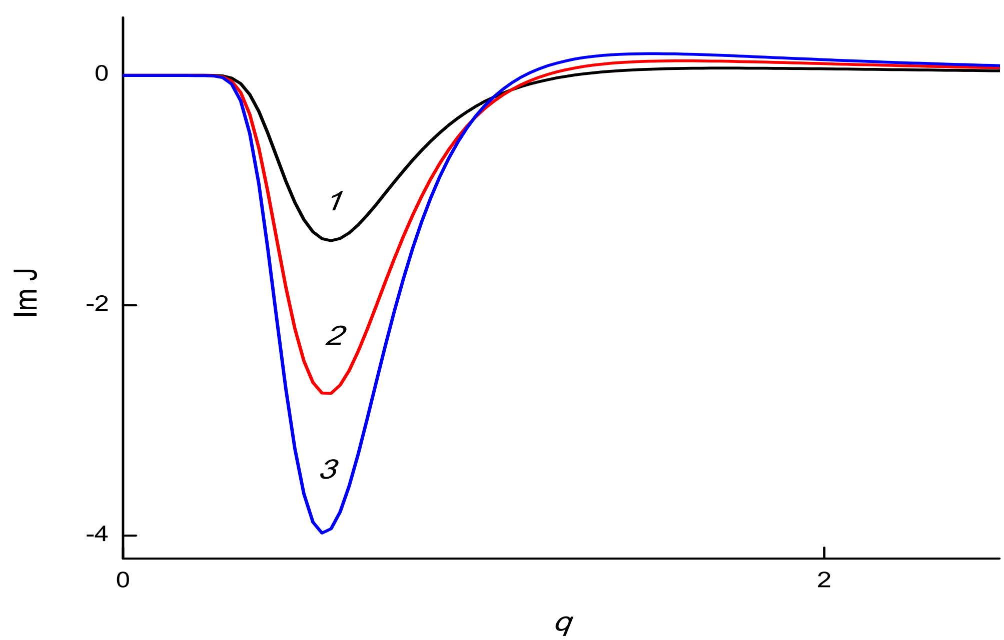

Fig. 2. Imaginary part of longitudinal electric current density, $\Omega = 1$. Curves $1, 2, 3$ correspond to values of dimensionless chemical potential $\alpha = -1, -0.3, +0.1$.

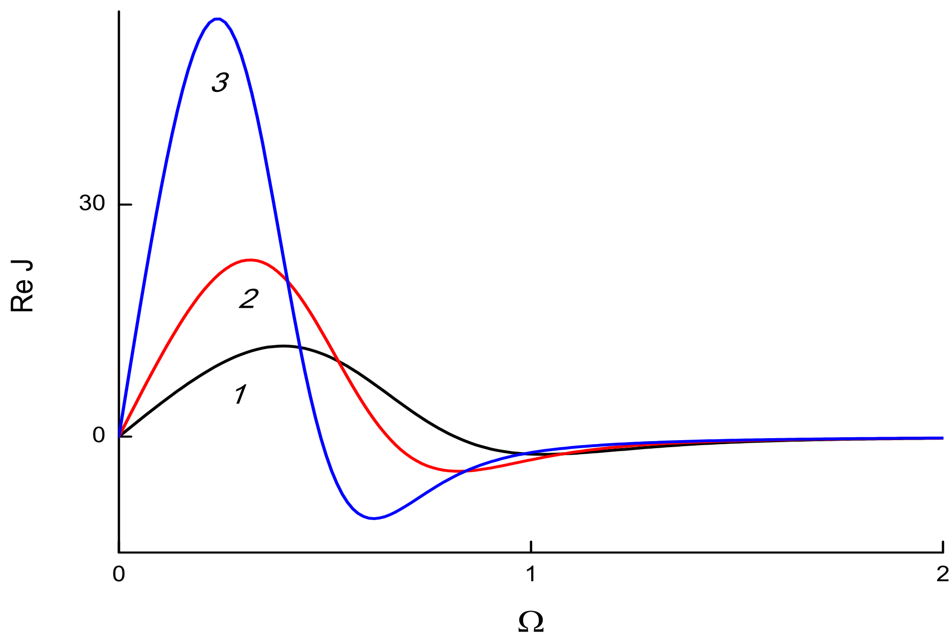

Fig. 3. Real part of longitudinal electric current density, $\alpha = 0$. Curves $1, 2, 3$ correspond to values of dimensionless wave number $q = 0.5, 0.4, 0.3$.

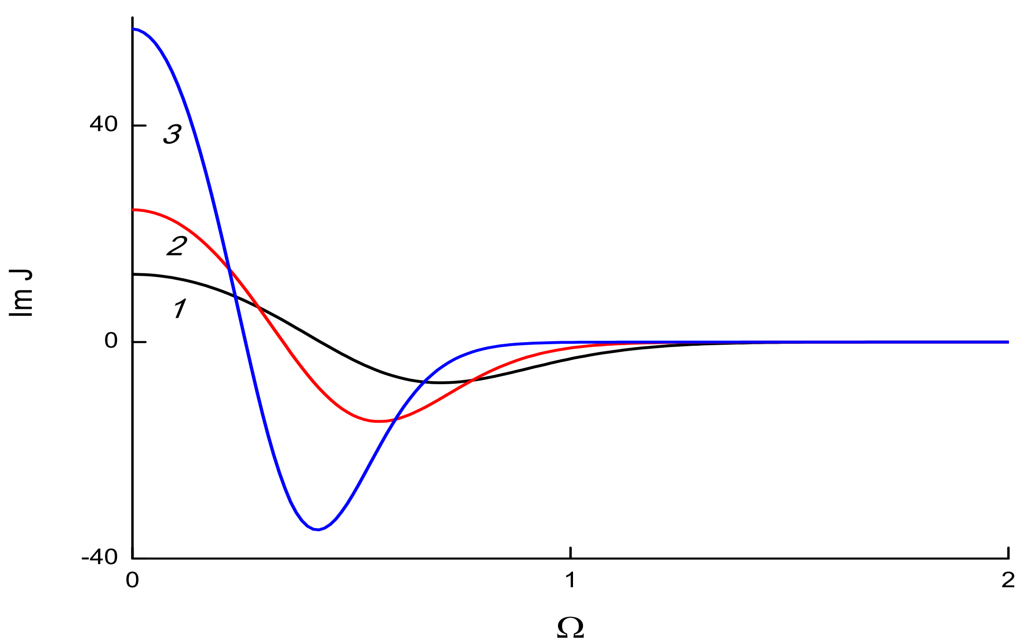

Fig. 4. Imaginary part of longitudinal electric current density, $\alpha = 0$. Curves $1, 2, 3$ correspond to values of dimensionless wave number $q = 0.5, 0.4, 0.3$.

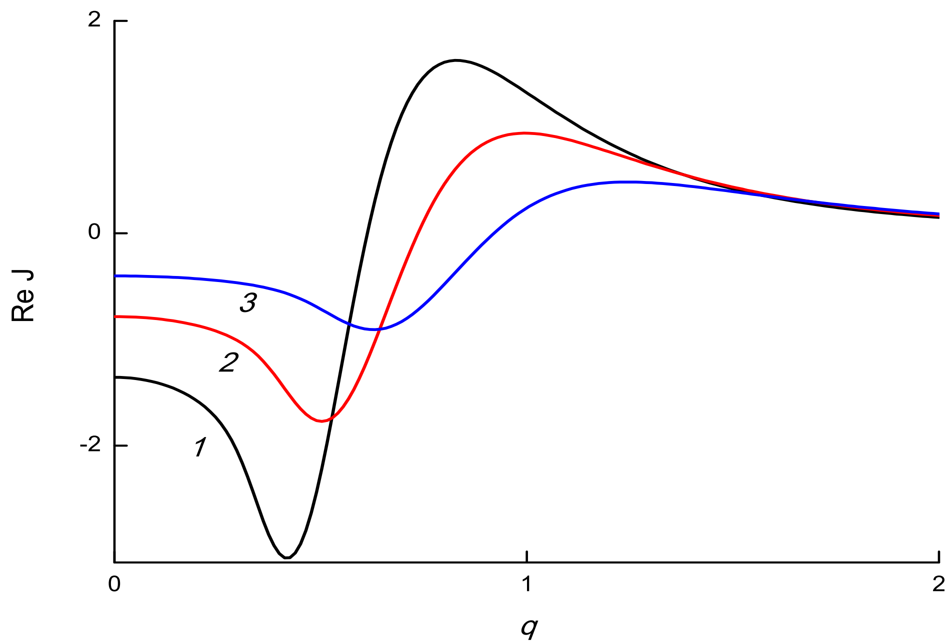

Fig. 5. Real part of longitudinal electric current density, $\alpha = 0$. Curves $1, 2, 3$ correspond to values of dimensionless frequency of electromagnetic field $\Omega = 1, 1.2, 1.5$.

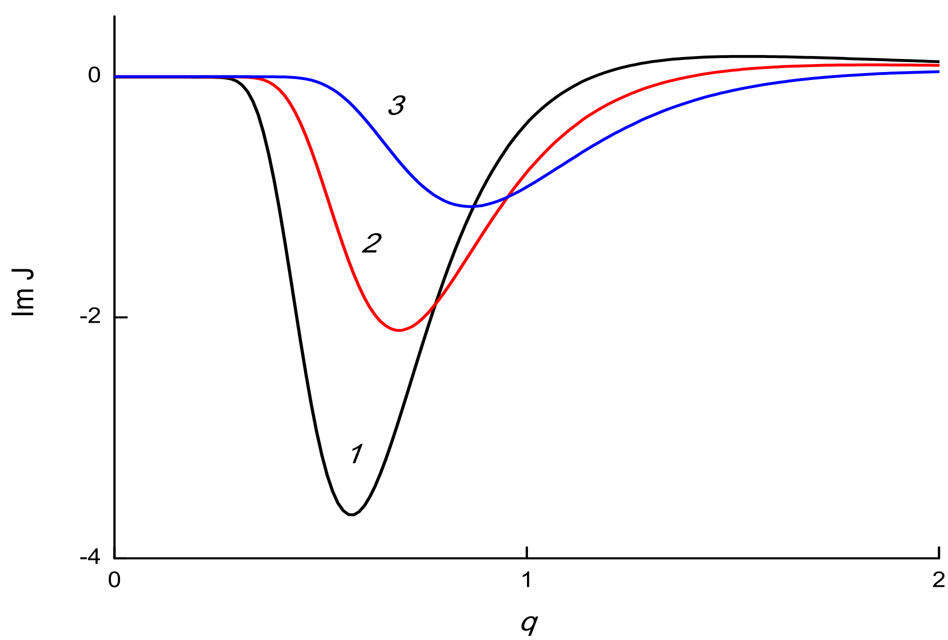

Fig. 6. Imaginary part of longitudinal electric current density, $\alpha = 0$. Curves $1, 2, 3$ correspond to values of dimensionless frequency of electromagnetic field $\Omega = 1, 1.2, 1.5$.